ESTIMATING MULTIPLE TRANSMITTER LOCATIONS FROM POWER MEASUREMENTS AT MULTIPLE RECEIVERS

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ABSTRACT

We consider the estimation of the locations of multiple transmitters based on received signal strength measurements at a network of randomly-placed receivers. We generalize the expectation-maximization (EM) method to create a quasi EM algorithm for localization under lognormal shadowing. Simulated performance is compared to a state-of-the-art global optimizer and to random guessing. Results reveal that the proposed quasi EM algorithm outperforms both alternatives in median and ninety-fifth percentile error, especially as the number of receivers increases.

Index Terms— transmitter localization, expectation-maximization, spectrum sensing, cognitive radio

1. BACKGROUND AND MOTIVATION

We consider the problem of using power measurements by multiple receivers to estimate the locations of multiple transmitters under lognormal shadowing, and propose a quasi expectation-maximization (EM) algorithm. Natural applications of this solution are those in which localization of a non-cooperative entity is required. For example, *uncoordinated* cognitive radio systems identify pockets of unused spectrum available for transmission, often called spectral holes, without cooperation from any legacy systems operating in the region. Recent work has argued that spectral hole identification can be significantly improved over simple detection-based approaches that allow a cognitive radio node to communicate only if the power it observes falls below a threshold [1, 2]. Estimating the locations of the legacy-system transmitters holds promise for increasing the degree to which cognitive radio nodes can exploit unused spectrum without causing harmful interference to legacy systems.

Transmitter localization for uncoordinated cognitive radio systems is more challenging than the standard problem of localization in wireless sensor networks, since it must be performed without any cooperation from or communication with the transmitters. When only one transmitter is present, its location can be determined from three received power measurements via trilateration or from a larger number via a least-squares estimate. However, when there are multiple transmitters contributing unknown proportions of the observed power at each receiver, the non-cooperative localization problem does not admit a straightforward solution.

In related work, Mark and Nasif have studied the transmitter localization problem and estimated the maximum interference-free transmit power when a single legacy transmitter is present [3]. Dogandzic and Amran [4] have derived an EM solution to the single transmitter localization problem under fading and shadowing, but even in that case the solution is highly complex, requiring multivariate numerical integration. In this paper, we focus on estimating the locations of multiple transmitters, and extend our previous work on doing so under additive white Gaussian noise (AWGN) [2, 5] to a more realistic lognormal shadowing model, which has been empirically validated as a model for the variations in received power due to obstacles in the signal path, which generally dominate the effects of additive noise [6]. As the joint distribution of the hidden and observed random variables in the lognormal model does not produce an analytic EM algorithm, we develop a quasi EM approach to multiple transmitter localization under lognormal shadowing.

2. SYSTEM MODEL

Let the unknown two-dimensional locations of the M transmitters be denoted by $\theta = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_M]^T \in \mathbb{R}^{M\times 2}$, where θ_i is the location of the ith transmitter. We assume that M is known and that all transmitters have the same constant transmit power P_0. We assume that the locations of the N receivers, $N \geq 2M$, are known but arbitrary. The problem is then to determine the maximum likelihood (ML) estimate $\hat{\theta}$ of θ based on the observed power measurements at the receivers: ideally, $\hat{\theta} = \arg\max_\theta p(r|\theta)$, where $r = [r_1 \ r_2 \ \ldots \ r_N]^T$

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and $r_j \in \mathbb{R}^+$ is the observed power at the jth receiver.

Let $d_j(\theta_i) \in \mathbb{R}^+$ denote the Euclidean distance from the transmitter located at θ_i to the jth receiver. Consider a log-distance path loss model such that the noise-free received power at the jth receiver from the ith transmitter is given by

$$S_{ij} = \rho P_0 \left(\frac{d_0}{d_j(\theta_i)} \right)^n, \tag{1}$$

where ρ is a constant that reflects the carrier frequency and antenna properties, n represents the path loss exponent, and d_0 is the close-in reference distance.

We assume that lognormal shadowing occurs independently for each transmitter-receiver pair. The resulting unknown measured power from the ith transmitter to the jth receiver is modeled as the random variable

$$H_{ij} = S_{ij} 10^{\frac{X_{ij}}{10}},$$

where $X_{ij} \sim \mathcal{N}(0, \sigma^2)$ is the gain in dB due to shadowing. The unknown measurements H_{ij} are related to r_j, the observed power at the jth receiver, by

$$r_j = \sum_{i=1}^{M} H_{ij}. \tag{2}$$

Finally, let the set of all $M \times N$ unknown measured powers be $H = [H_1\ H_2 \ldots H_M]$, where $H_i = [H_{i1}\ H_{i2} \ldots H_{iN}]$.

3. ITERATIVE LOCALIZATION TECHNIQUE FOR LOGNORMAL SHADOWING

Under independent lognormal shadowing, the likelihood function of H conditioned on transmitter locations θ is the product of MN lognormal densities:

$$p(H = h|\theta) = \prod_{i=1}^{M} \prod_{j=1}^{N} \frac{10 \log_{10} e}{h_{ij} \sigma \sqrt{2\pi}} e^{-\frac{(10 \log_{10}(h_{ij}) - 10 \log_{10}(S_{ij}))^2}{2\sigma^2}}.$$

From (2), the observed power r_j is the sum of M lognormal random variables. No analytic distribution exists for the sum of lognormal random variables, and hence no closed-form expression for $E[H|r, \hat{\theta}]$ can be obtained, which complicates the expectation step of the EM algorithm.

We have designed a quasi EM approach, which avoids the conditional likelihood computation required by the true EM approach, and notably does not require any knowledge or estimation of the shadowing variance. The algorithm alternates between (a) estimating each transmitter location independently based on an allocated percentage of the power received at each receiver; and (b) allocating a percentage of the power received at each receiver to each transmitter proportional to the expected received power given the last transmitter location estimates. This approach is not limited to lognormal shadowing; in fact, it can be applied to any stochastic model.

Step 1: Randomly generate initial estimates $\hat{\theta}$ of the M transmitter locations.

Step 2: Given M estimated transmitter locations $\hat{\theta}$, determine the expected log-power (in dB) at the jth receiver from the ith transmitter for $i = 1$ to M and $j = 1$ to N:

$$\begin{aligned} e_{ij} &= E[10 \log_{10} H_{ij}] = E\left[10 \log_{10} \left(S_{ij} 10^{\frac{X_{ij}}{10}} \right)\right] \\ &= 10 \log_{10}(\rho P_0) + 10n \log_{10} \left(\frac{d_0}{d_j(\hat{\theta}_i)} \right). \end{aligned}$$

Step 3: Normalize the expected log-power values e_{ij} so that they give a total power at each receiver equal to the observed power at that receiver:

$$\tilde{e}_{ij} = 10 \log_{10} \left(\frac{r_j 10^{\frac{e_{ij}}{10}}}{\sum_i 10^{\frac{e_{ij}}{10}}} \right).$$

Note that this normalization is proportional, rather than the additive normalization prescribed by the EM algorithm for AWGN [5]. We chose to use proportional normalization based on preliminary results and to avoid situations where an additive correction results in a negative power.

Step 4: Using the expected log-power values $\tilde{e}_{ij}$, re-estimate the transmitter locations by minimizing the sum of squared dB error:

$$\hat{\theta}_i = \arg\min_{\tilde{\theta}_i} \sum_{j=1}^{N} \left(\tilde{e}_{ij} - 10 \log_{10} \left(\frac{\rho P_0 d_0^n}{d_j(\tilde{\theta}_i)^n} \right) \right)^2.$$

Note that minimizing the sum of squared dB power error is intuitively pleasing under lognormal shadowing, since the power at receiver j due to transmitter i is a Gaussian random variable in the log domain, and hence squared error is inversely proportional to likelihood.

Step 5: If the stopping criterion is not yet met, return to Step 2.

In our simulations, we stop after a fixed number of iterations, but other convergence measures are of course possible.

To increase the likelihood that the global minimum of the cost function is reached, the quasi EM algorithm is run to convergence multiple times with different sets of random initial conditions. The final estimate of the transmitter locations is chosen to be the solution that yields the lowest sum-of-squared log-power errors:

$$C(\hat{\theta}) = \sum_{j=1}^{N} \left(\log_{10} \left(\frac{r_j}{\sum_{i=1}^{M} \left(\frac{\rho P_0 d_0^n}{d_j(\hat{\theta}_i)^n} \right)} \right) \right)^2. \tag{3}$$

4. EXPERIMENTS

We compare the performance of the proposed quasi EM technique to *constriction particle swarm optimization* (CPSO) and to random guessing. CPSO is a variant of particle swarm optimization that introduces a constriction factor that reduces the velocity of the particles as the search iterates, thereby increasing the likelihood that the particle will converge to a minimal location [7]. The CPSO particles search the $(M \times 2)$-dimensional θ-space to directly minimize the sum-of-squared log-power errors cost function, given by (3), that we use to compare results from different initial conditions for the quasi EM algorithm. We use the choices for the inertial weight α, cognitive scaling parameter β, and social scaling parameter γ recommended by Schutte and Groenwold [7].

In our implementation of CPSO, each particle's initial velocity is drawn from a uniform distribution on the interval $[-0.5, 0.5]$ times the width of the search area, while its initial location is chosen using the following "smart" technique, which is repeated for each particle. First, the k-means algorithm is run with random initial centroids to group the N receivers into M clusters based on geographic proximity. Then, we assume that all of the power observed at the receivers in a given cluster is generated by a single transmitter, and find the least-squares estimate of that transmitter's location. The resulting set of M transmitter locations constitutes the "smart" initial location for the particle in the CPSO search space. A more detailed description of smart initial conditions can be found in [2].

4.1. Experimental Setup

The region of interest is assumed to be a one-kilometer square, with certain constraints on the transmitter and receiver geometries. First, transmitters are assumed to be separated by at least 200 meters, reflecting the physical reality that primary transmitters using the same frequency band would interfere if they were too close together. Second, all receivers are assumed to be at least twice the reference distance d_0 from all transmitters, a constraint required to guarantee that the log-distance propagation model yields realistic results [6].

These placement constraints are reflected in the implementation of the proposed quasi EM algorithm. While the algorithm is iterating, if any transmitter location estimates are within 100 meters of each other, or if any single estimate is more than 50% (500 meters) outside the square region of interest, the algorithm is assumed to be converging incorrectly and the problematic transmitter(s) are uniformly randomly reassigned to new locations in the square region. (The proximity threshold is 100 meters instead of 200 meters so that transmitter location estimates can temporarily move slightly closer as the algorithm converges.) Once the algorithm stops iterating, any location estimates that lie outside the square region are clipped to its nearest boundary. In contrast, the locations of the CPSO particles are clipped in order to keep them within the search area at all times.

The quasi EM algorithm is run M^2 times with different uniformly randomly drawn initial estimates $\hat{\theta}$, with the number of iterations (for each initial estimate) fixed at 10. For consistency, CPSO uses M^2 particles, generated via the previously described "smart" technique, and is allowed 10 iterations per particle for a total of $10M^2$ guesses. Similarly, the random guessing approach makes $10M^2$ random guesses (uniformly drawn from the search space) of the transmitter locations and chooses the random guess that minimizes the sum-of-squared log-power error cost function given in (3).

4.2. Results

The simulated performance of the quasi EM, CPSO, and random guessing algorithms is presented in Figs. 1 and 2. The chosen performance metric is the average squared distance error between estimated and true transmitter locations, where the average is taken over the transmitters whose locations are estimated. The values have been normalized to assume a square of unit area. Performance figures show both the median and 95^{th} percentile error over 1000 different random draws of $M = 3$ transmitters and $N = 6$ to $N = 40$ receivers with shadowing variance of $\sigma^2 = 4$ and $\sigma^2 = 16$. The median error reflects each algorithm's typical performance, while the 95^{th} percentile error gives insight into the severity of erroneous estimates for each algorithm.

The quasi EM localization approach produces the smallest median error across all values of N and σ^2 considered, outperforming both CPSO and random guessing. This performance gap increases as N increases from its minimum value, especially for larger shadowing variance. Median error begins to flatten near $N = 20$ for $\sigma^2 = 4$ but continues to drop as N increases to 40 when $\sigma^2 = 16$. As the number of receivers grows large, additional power measurements are less likely to provide independent information, and hence the resulting performance improvements are not as significant. When the shadowing variance is large, however, a large number of measurements are necessary to overcome the severe shadowing effects. When only a small number of receivers is available, each receiver can take measurements at multiple locations. Quasi EM's avoidance of severely erroneous estimates relative to competing schemes, especially for sufficiently large N, is particularly noteworthy.

5. CONCLUSIONS AND OPEN QUESTIONS

We have shown that a quasi EM approach can be effective in estimating multiple transmitter locations from multiple power measurements when propagating signals experience lognormal shadowing. Compared to a state-of-the-art global optimization method and to random guessing, the proposed quasi EM algorithm produces roughly one-half to one-tenth the er-

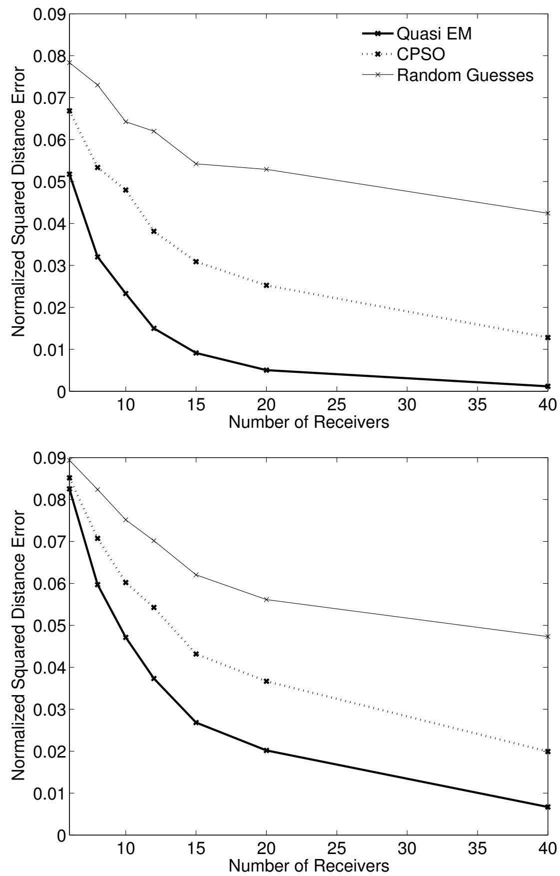

Fig. 1. Median normalized average squared distance error for $\sigma^2 = 4$ (top) and $\sigma^2 = 16$ (bottom).

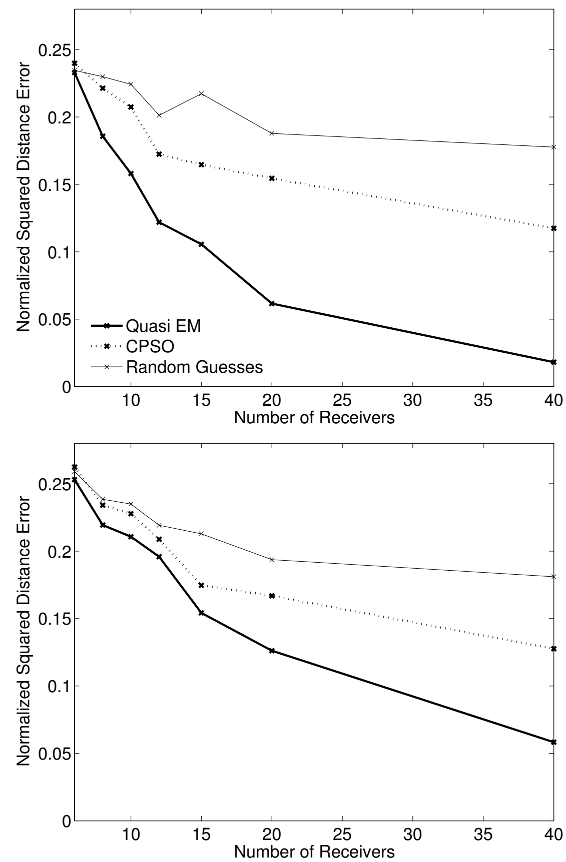

Fig. 2. 95^{th} percentile normalized average squared distance error for $\sigma^2 = 4$ (top) and $\sigma^2 = 16$ (bottom).

ror when given the same number of guesses. The practical complexity of these estimation methods depends on the implementation architecture, but even random guessing requires evaluating a cost function to select a best random guess. Thus we argue that these three estimation methods are comparable on a per-guess basis, though quasi EM does require a more complex procedure to form each guess.

Future work should explore how the sum-of-log-power-errors cost function (3) that one can minimize in practice relates to the final cost function, the sum of distances between the estimated and true transmitters. Additionally, we assumed that the number of transmitters and their transmit powers are known; how to extend the estimation algorithms to model these quantities as random variables is an open question.

6. REFERENCES

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