Estimation of Position from Multistatic Doppler Measurements

Evan Hanusa
Electrical Engineering
University of Washington
Seattle, WA, USA
hanusaem@ee.washington.edu

David Krout
Applied Physics Laboratory
University of Washington
Seattle, WA, USA
dkrout@apl.washington.edu

Maya R. Gupta
Electrical Engineering
University of Washington
Seattle, WA, USA
gupta@ee.washington.edu

Abstract – *We implement and evaluate a method infer position from Doppler measurements in a multistatic sonar scenario and present a likelihood approach for doing so. Doppler measurements are used to create likelihood surfaces for each of the transmitter-receiver pairs. The likelihood surfaces are combined and can then be used as-is or combined with additional position measurements. The final likelihood surface is usable in a Bayesian-style tracker or can be used to estimate position of a contact for use in a contact-based tracker. We show how the estimate improves with the addition of multiple receivers and show how the use of Doppler information can improve tracking results.*

Keywords: Doppler, likelihood, estimation, tracking.

1 Introduction

In a multistatic sonar environment, a single Continuous Wave (CW) waveform transmission yields a measurement of bistatic range r, bearing b, and bistatic Doppler d for each contact that is within range of any receiver. It is standard to use bistatic range and bearing measurements to estimate the position of the contact. Bistatic Doppler is the average of the target's velocity v towards the transmitter and towards the receiver. This allows a single measurement of bistatic Doppler to estimate the target's velocity.

In this paper, we propose the inference of position from multiple bistatic Doppler measurements and propose a likelihood approach for doing so. The method is based on the likelihood surfaces used in Bayesian-style trackers as described by Stone [1]. The likelihood surfaces created from each of the bistatic Doppler measurements are combined, allowing for an improved estimate of position. Section 2 will contrast our usage of Doppler information to other work with similar measurements. Section 4 will give a brief motivation for our approach under simple conditions. Section 5 will derive the estimation method. Section 6 will discuss the performance of the position estimation under different numbers of receivers and show results in a tracking framework. Section 7 will discuss the overall results and suggest future research directions.

2 Related Work

Doppler measurements are available in a variety of sonar and radar scenarios and configurations. In passive sonar or radar, tonal frequencies can be tracked to obtain an estimate of Doppler, and then the Doppler and bearing information can be used to track a target [2, 3, 4]. Recently it has been shown that under various conditions (stationary transmitter, for example) moving receivers can apply a signal processing-based approach to improve the estimate of target position provided that the entire raw signal can be transmitted [5]. Other techniques use a mean-square error fitting technique to track a CW-radiating projectile over time [6]. These methods all require a series of passive measurements taken over time to localize or track a target. Active sonar configurations allow for estimation of range and bearing to target, and these are directly used to either estimate position or create a likelihood surface for use in a Bayesian-style tracker. Wang et al. show how Doppler can be combined with an estimate of position to improve the initial estimate of velocity in a tracker [7]. Also, Doppler can be used as a feature in classification to potentially discard contacts as clutter. La Cour uses Doppler measurements in a Bayesian tracking framework, however our work differs in that we attempt to estimate position from multiple measurements at a single time [8].

Our work focuses on the use of Doppler in a multistatic, active sonar configuration. Each receiver will measure the bistatic Doppler shift of the signal, and these measurements of Doppler will be combined into a single likelihood surface that can be used to form estimates of position (and/or velocity), and combined with likelihood surfaces from bearing and bistatic range measurements.

3 Term Definition

We define the following terms:

d_i : measurement of Doppler for receiver i
c : contact position; $[x, y]$
v : contact velocity; $[dx, dy]$

t : ping transmission location; $[x, y]$
r_i : receiver i location; $[x, y]$
$\hat{c}$: estimate of contact position
$\vec{u}_{tc}$: unit vector from transmitter to contact
$\vec{u}_{r_ic}$: unit vector from receiver i to contact
$L()$: a likelihood surface
$P()$: a probability

4 Motivation

In a multistatic active sonar geometry, every receiver r_i will obtain a measurement of bistatic Doppler, d_i on each transmitted ping. Each measurement of d_i is the average of components of the contact's velocity v in the direction of the transmitter t and the receiver r_i. Under the assumption that the transmitter and receiver are stationary, Equation 1 is the vector form of the bistatic Doppler equation:

$$d_i = \frac{v^T}{2} \left(\vec{u}_{tc} + \vec{u}_{r_ic} \right), \tag{1}$$

where $\vec{u}_{tc}$ and $\vec{u}_{r_ic}$ are the unit vectors in the from the transmitter and receiver to the contact as defined in Equation 2:

$$\begin{aligned} \vec{u}_{tc} &= \frac{t - c}{\|t - c\|_2} \\ \vec{u}_{r_ic} &= \frac{r_i - c}{\|r_i - c\|_2}. \end{aligned} \tag{2}$$

Even if the velocity is known, the unit vectors are underconstrained in Equation 1. Given d_i, the position of the contact is restricted to a curve. A second simultaneous Doppler measurement similarly constrains the position to a different curve. The contact must lie on one of the intersections of the two curves. Figure 1 gives a simple example, where both the velocity and Doppler are known exactly.

Note that the two curves still overlap at multiple points. Addition of more receivers would remove this ambiguity. In a scenario where the velocity is not known exactly, we treat the velocity as a random variable with a known probability distribution. With a distribution on velocity, each receiver has an area in which the contact must lie. When multiple measurements with uncertainty are combined, the contact must lie in the overlapping area. Figure 2 shows four likelihood surfaces for four different receivers of a single transmitted ping. When the likelihood surfaces are combined in the (x, y, dx, dy) space, the true contact location (the white dot) has a high likelihood, as shown in Figure 3. In the next section, we will derive the equation for the likelihood surface in the more realistic case that there is also uncertainty in the Doppler measurements (e.g. due to noise) and contact velocity.

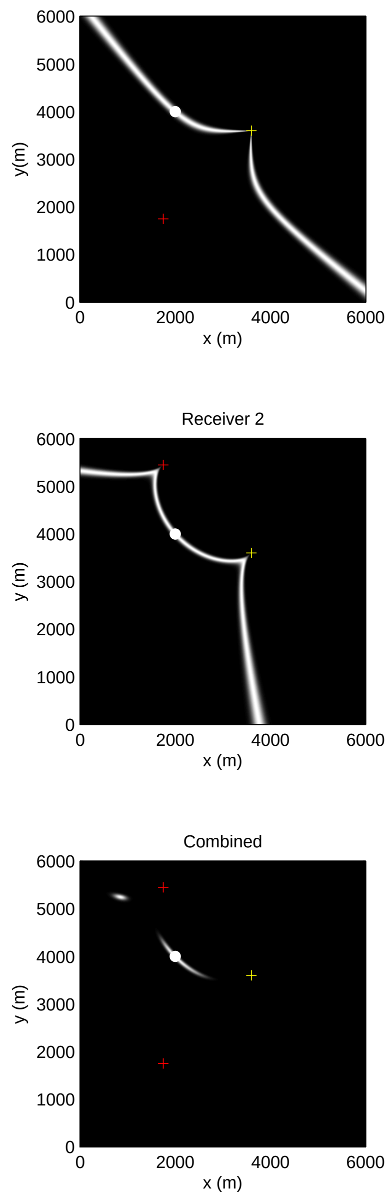

Figure 1: A simple example with known velocity and Doppler, where the white lines indicate possible locations of the target. The top two subfigures show the possible locations of the target (white circle) given a measurement of Doppler at a receiver (red plus). Note the different receiver locations in the top two figures. The bottom subfigure shows the possible locations of the target when the two measurements are combined.

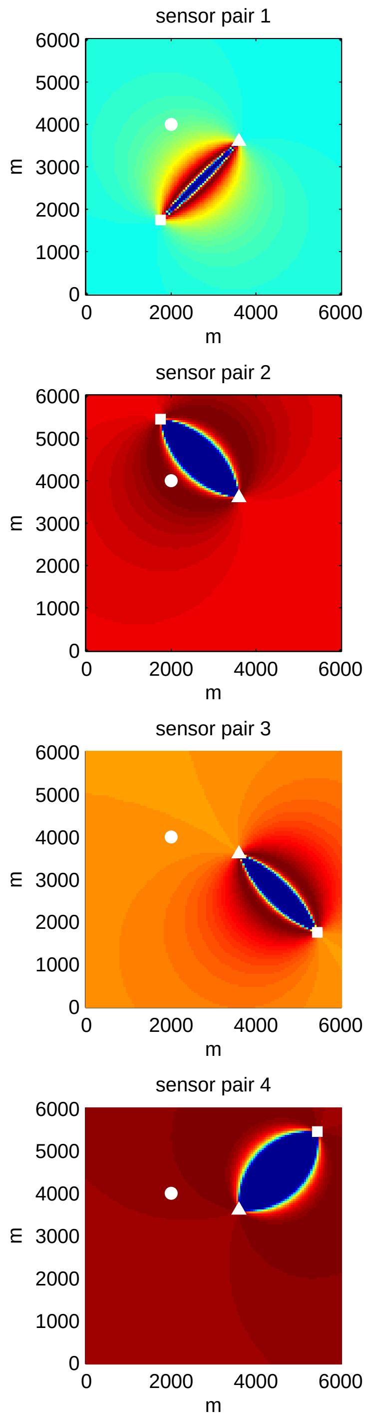

Figure 2: Four individual likelihood surfaces, with a uniform prior on target velocity, and $\sigma_d = 0.1$. The transmitter is centrally located, indicated by the white triangle. Four different likelihood surfaces are formed using receivers at different locations marked by the white squares. The contact is marked with a white circle.

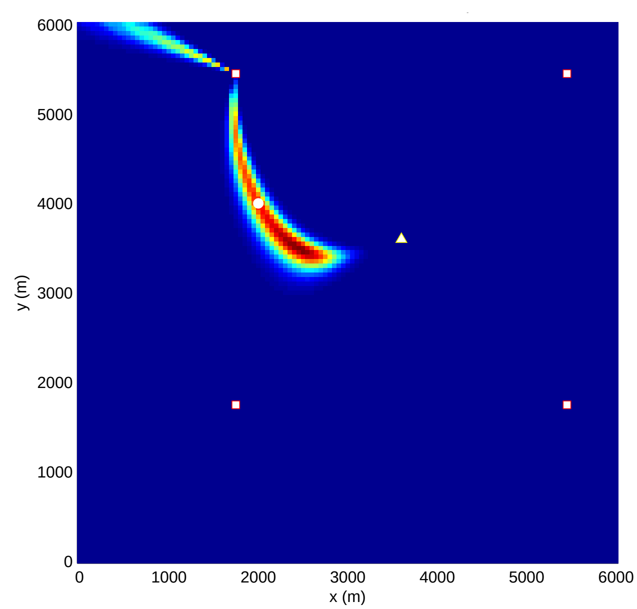

Figure 3: The position likelihood surface formed when the surfaces shown in Figure 2 are combined in the (x, y, dx, dy) state space.

5 Likelihood Surface and Position Inference

Given n independent Doppler measurements a likelihood function can be formed over the state space $[c, v]$. Given n Doppler measurements, the position and velocity likelihood function is defined as follows:

$$L(c, v|d_{1...n}, t, r_{1...n}) = P(c, v|d_{1...n}, t, r_{1...n}). \quad (3)$$

By Bayes' rule:

$$L(c, v|d_{1...n}, t, r_{1...n}) = \gamma^{-1} P(d_{1...n}|t, r_{1...n}, c, v) P(c, v|t, r_{1...n}), \quad (4)$$

where γ is the normalizing constant:

$$\gamma = P(d_{1...n}|t, r_{1...n}). \quad (5)$$

Assuming the errors on the Doppler measurements are independent,

$$L(c, v|d_{1...n}, t, r_{1...n}) = \gamma^{-1} \left(\prod_{i=1}^{n} P(d_i|t, r_i, c, v) \right) P(c, v|t, r_{1...n}). \quad (6)$$

Integrating out velocity gives the contact position likelihood surface,

$$L(c|d_{1...n}, t, r_i) = \gamma^{-1} \int \prod_i P(d_i|t, r_i, c, v) P(c, v|t, r_{1...n})\, dv. \quad (7)$$

The prior, $P(c, v|t, r_{1...n})$ in Equation 7, can be uniform when no information about the contact's state is available, can take into account any other available measurements (such as range or bearing), or can include kinematic information from a tracker. One of the main motivations for this method is that you will always be able to constrain the velocity prior - there is no need to consider velocities greater than the maximum possible velocity of your target.

We assume iid additive zero mean Gaussian error on the Doppler measurements, resulting in the likelihood function,

$$L(c|d_{1...n}, t, r_i) = \gamma^{-1} \int \prod_i \mathcal{N}\left(\frac{v^T}{2}(\vec{u}_{tc} + \vec{u}_{r_i c}); d_i, \sigma_d^2\right) P(c, v)\, dv. \quad (8)$$

If a single estimate of position is necessary, rather than the likelihood surface, the estimate of the contact's position is simply

$$\arg\max_c L(c|d_{1...n}, t, r_i) = \arg\max_c \int \prod_i P(d_i|t, r_i, c, v) P(c, v)\, dv. \quad (9)$$

At first glance, Equation 8 appears to be readily analyzable (or possibly simplifyable) using the product of Gaussian rule. However, recalling that the terms $\vec{u}_{tc}$ and $\vec{u}_{r_i c}$ both have L_2 norms in the denominator that are functions of c, one finds that in general the non-linearity of the bistatic Doppler equation makes an analytic solution intractable. To evaluate the performance of our Doppler position estimation technique, we will sample the likelihood surface in the state space (x, y, dx, dy) and compute the results using numeric techniques. The approach taken is described in the next sections.

6 Simulations

The following simulations were designed to evaluate the performance of combining Doppler measurements in position estimation of a single ping and also how the inclusion of Doppler affects tracking in a multistatic scenario. In practice, these estimates are combined with the independent information provided by contact position estimation, and these simulations are designed to understand how much and what kind of information multistatic Doppler can add. In both of the simulations, we create a grid over the state space and calculate the likelihood at each of the grid locations. The position state is uniformly sampled in x and y. Velocity is uniformly sampled in $|v|$ and $\angle v$.

6.1 Position Estimation Simulation

This simulation is designed to evaluate the effectiveness of using Doppler alone to estimate position. The contact is placed at the center of a circle of radius R. A transmitter is placed directly 'north' of the contact. Receivers are uniformly spaced around the circle with a uniformly random rotation. The target is assigned a random velocity with $|v| \leq v_{max}$. Figure 4 shows the a single random instance of the simulation.

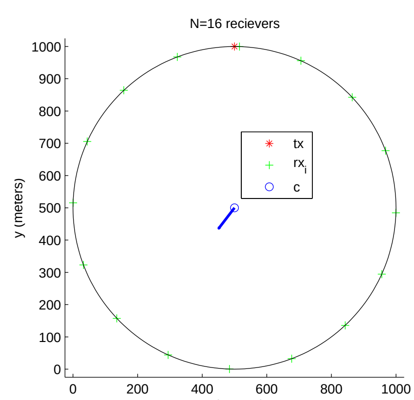

Figure 4: One instance of a random simulation with 16 equally spaced receivers, centered around the target. The transmitter is located directly north of the target. The blue line is proportional to the target's velocity.

Several metrics were used to evaluate different properties of the position estimation, and each of the metrics was averaged over 1500 random runs. The likelihood surface was normalized to sum to one (making it a probability mass function), allowing comparison between simulations with different numbers of receivers. The likelihood of the true contact position and the entropy, H, is calculated for each number of receivers.

$$H = -\sum_{x=x_{min}}^{x_{max}} \sum_{y=y_{min}}^{y_{max}} p_{(x,y)} \log p_{(x,y)}, \quad (10)$$

where $p_{(x,y)}$ is the value of the normalized likelihood surface at contact position (x, y).

The distance from the maximum of the likelihood surface to the true contact location was used to evaluate the accuracy of the maximum likelihood position estimate.

6.2 Position Estimation Results

Simulations were run with the following parameters: $N = [2, 3, 4, 8, 16, 32]$ receivers, $R = 500m$, $x_{step} = y_{step} = 10m$, $v_{max} = 5m/s$, $|v|_{step} = 0.2m/s$, $\angle v_{step} = 10°$, $\sigma_d = 0.5m/s$.

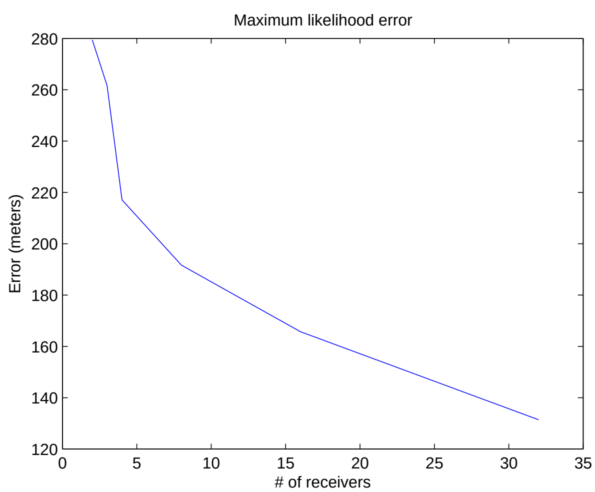

Figure 5: Average error from true position to maximum likelihood estimate of position.

Figure 5 shows the mean error between the true contact position and the maximum likelihood position. Figure 6 shows the normalized likelihood of the true position, and the entropy of the likelihood surface as the number of receivers increases.

As the number of receivers increases, the likelihood of the true contact position increases and the entropy decreases. This is the desired result: decreasing entropy means that the likelihood surface is becoming more peaked. The increasing likelihood of the true position is useful in scenarios where additional information is know about position (bearing or range, for example).

6.3 Tracking Scenario

We propose the following simulation to evaluate the impact of including Doppler-based position estimation in a Kalman filter based tracker. A target is moving in a semi-circle through an array of receivers, with a single transmitter in the center of the scenario. Interping interval is $3m$, and the target is moving with $|v| = 3.491m/s$. There are a total of 41 pings in each simulation. Each transmitted ping results in a detection at all receivers ($p_D = 1$), and there are no false detections (clutter). The bearing b, bistatic range r, and Doppler d measurements are corrupted with zero mean additive Gaussian noise: $\sigma_b = 5°, \sigma_r = 600m, \sigma_d = 0.5m/s$. The Kalman filter underlying the tracker is a nearly-constant-velocity model. Note that this means the tracker will not be able to correctly predict the next position due to the constant velocity assumption being violated. This is by design to ensure that the tracker is relying on the maximum likelihood position estimates.

Figure 7 shows all the measurement data from a single random simulation, with the true track marked in red.

To combine the contacts from all receivers, we use Equation 8. The bearing and bistatic range information is incorporated in the prior, $p(c, v)$:

$$p(c, v) = \prod_{i=1}^{N} \mathcal{N}(b_{(x,y,i)}; b_i, \sigma_b^2)\mathcal{N}(r_{(x,y,i)}; r_i, \sigma_r^2), \quad (11)$$

where $b_{(x,y,i)}$ is the bearing from receiver i to point c (x, y), $r_{(x,y,i)}$ is the bistatic range from receiver i to point c, b_i is the bearing measurement, and r_i is the bistatic range measurement.

The likelihood function is evaluated over a grid of (c, v), and the maximum likelihood position is the measurement that is passed to the tracker. The likelihood function was evaluated at $x_{step} = y_{step} = 100m, \angle v_{step} =$

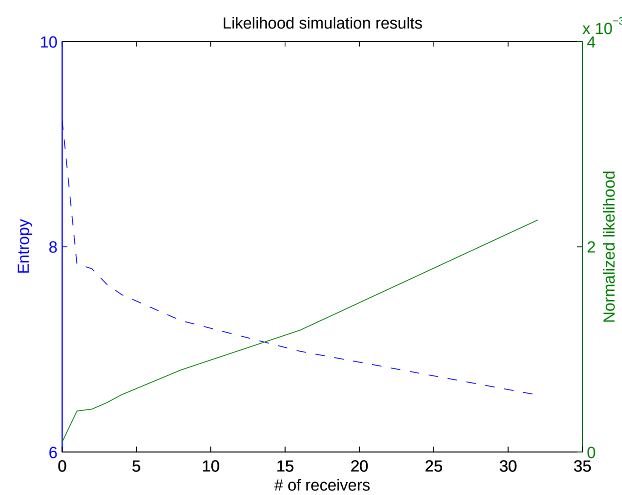

Figure 6: Entropy (dashed blue line) and likelihood of true contact (solid green line) for increasing number of receivers.

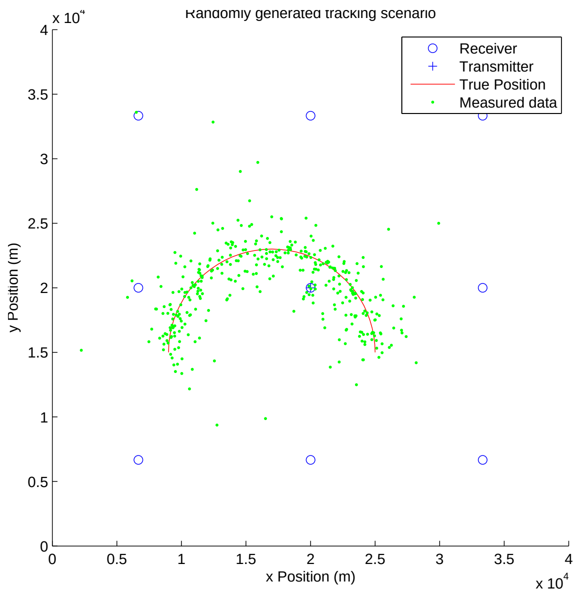

Figure 7: Measurement data from a single random tracking scenario.

$4.5°, |v|_{max} = 5m/s, |v|_{step} = 0.25$. The same process is repeated using only Doppler information (uniform prior $p(c, v)$) and without Doppler information (the likelihood surface is proportional to Equation 11).

6.4 Tracking Results

To evaluate the impact that the inclusion of Doppler information is having, 400 randomizations of the simulation were run, and the resulting tracks averaged. In this case, there is only one target and no clutter, so the tracker is simply a Kalman Filter which smooths the position data.

To compare the accuracy of the BRD (Bearing, Range, and Doppler), BRD with filter, D (Doppler alone), BR (Bearing and Range) and BR with filter, we use the mean squared error of the entire track of length T as calculated in Equation 12.

$$TSE_n = \sum_{t=1}^{T} \|truePos_t - estPos_t\|_2^2$$
$$MTSE = \frac{1}{N}\sum_{n=1}^{N} TSE_n, \tag{12}$$

where TSE_n is the Track Squared error for a single track n and the TSE is averaged over $N = 400$ independent runs.

Results are shown in table 1 for two different values of σ_d. Note that a decrease in σ_d results in an improved MTSE.

Figure 8 shows the results for a run of the tracker on the random scenario in Figure 7. The unfiltered bearing and range maximum likelihood estimates (BR MLE) and the filtered BR MLE estimates are in blue. The unfiltered bearing, range, and Doppler MLE (BRD MLE) estimates are in black. It is immediately apparent that at this level of Doppler measurement noise, Doppler information alone is not sufficient for position estimation. The results with BRD and BR are very similar, and for a single randomization it is difficult to compare the two. The ML estimates from both the BR and BRD appear to be converging to the true track, suggesting that the ML estimator is unbiased in both cases. The filtered data is biased outside the true track, due to the incorrect nearly constant velocity assumption of the Kalman filter. The Doppler-only position data is high variance.

σ_d	BR	BRD	D	BR Filt	BRD Filt
0.5	2.85e6	**2.76e6**	7.46e8	1.65e6	**1.60e6**
0.2	2.84e6	**2.39e6**	1.46e8	1.62e6	**1.42e6**

Table 1: MTSE (m^2) for position estimation with and without the tracker's Kalman filter. The inclusion of Doppler information into the position estimate improves the performance with and without the Kalman filter.

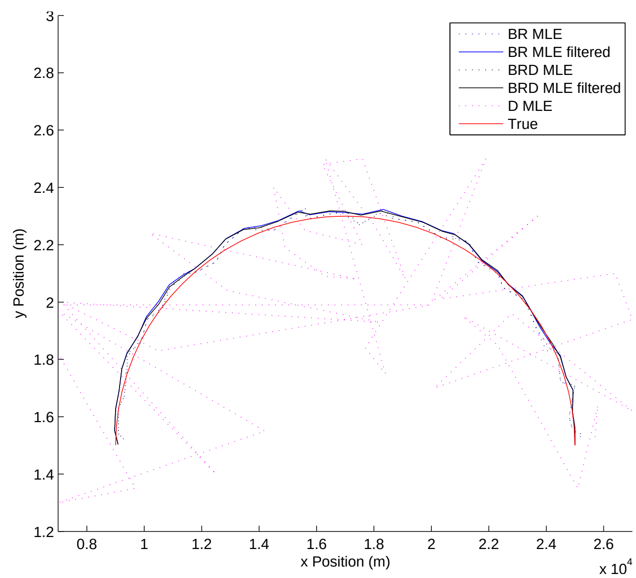

Figure 8: Results from a single tracking run, $\sigma_d = 0.5$.

7 Conclusions and Future Work

In this paper we began to study whether multistatic Doppler contains orthogonal information about position than the bearing and range, and whether multistatic Doppler data can be used to add value to position estimates. The estimate of position could be used on its own or appropriately combined with other independent information.

Using Doppler as the sole source of information yields a poor estimate of position. Once included with other position information (bearing, bistatic range), it improves the accuracy both of single-ping position estimation and of tracking results using a simple Kalman filter slightly in the simulations we ran. We plan to investigate further through simulations in what conditions the addition of Doppler measurements will be most valuable, and how much value it can add.

In our approach, we chose to use Doppler only to gain an estimate of position, however it is straightforward to use this technique to estimate velocity as well. The likelihood functions would be derived in the same way, and rather than integrating over velocity, the $\arg\max_{c,v}$ would be taken of Equation 7.

Additionally, the Doppler estimate of position could incorporate information from the tracker, in the form of a prediction of the current target position and velocity. Once incorporated into the prior in Equation 9, the performance of this estimator would likely improve.

Acknowledgment

This work was funded by the U.S. Office of Naval Research, Contract Number N00014-01-G-0460.

References

[1] L.D. Stone, T.L. Corwin, and C.A. Barlow. *Bayesian multiple target tracking*. Artech House, Inc. Norwood, MA, USA, 1999.

[2] Y.T. Chan and J.J. Towers. Passive localization from Doppler shifted frequency measurements. *Acoustics, Speech, and Signal Processing, IEEE International Conference on*, 40(10):1465–1468, 1991.

[3] Y.T. Chan and S.W. Rudnicki. Bearings-only and Doppler-bearing tracking using instrumental variables. *Aerospace and Electronic Systems, IEEE Transactions on*, 28(4):1076 –1083, Oct 1992.

[4] Y.T. Chan and J.J. Towers. Sequential localization of a radiating source by Doppler-shifted frequency measurements. *Aerospace and Electronic Systems, IEEE Transactions on*, 28(4):1084 –1090, Oct 1992.

[5] A. Amar and A.J. Weiss. Localization of narrowband radio emitters based on Doppler frequency shifts. *Signal Processing, IEEE Transactions on*, 56(11):5500 –5508, Nov. 2008.

[6] E. Weinstein and N. Levanon. Passive array tracking of a continuous wave transmitting projectile. *Aerospace and Electronic Systems, IEEE Transactions on*, AES-16(5):721 –726, Sept. 1980.

[7] X. Wang, D. Musicki, R. Ellem, and F. Fletcher. Enhanced Multi-Target Tracking with Doppler Measurements. *Information, Decision and Control, 2007. IDC'07*, pages 53–58, 2007.

[8] Brian R. La Cour. Bayesian sensor registration for multistatic active sonar. *Conference Proceedings, IEEE OCEANS '05 EUROPE, 20-23 June 2005, Brest, France*, pages 131–136.