A Quasi EM Method for Estimating Multiple Transmitter Locations

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***Abstract*—We consider estimating multiple transmitter locations based on received signal strength measurements by a sensor network of randomly located receivers. This problem is motivated by the search for available spectrum in cognitive radio applications. We create a quasi expectation maximization (EM) algorithm for localization under lognormal shadowing. Simulated performance is compared to random guessing and to global optimization using constriction particle swarm (CPSO). Results show that the proposed quasi EM algorithm outperforms both alternatives given a fixed number of guesses, and the performance gap grows as the number of transmitters increases.**

***Index Terms*—transmitter localization, expectation-maximization, spectrum sensing, cognitive radio, sensor network, particle swarm optimization**

I. Background and Motivation

We propose a quasi expectation-maximization (EM) algorithm to address the problem of estimating the locations of multiple transmitters based on power measurements at multiple receivers under lognormal shadowing. Natural applications of this solution are those in which localization of a non-cooperative entity is required. For example, uncoordinated cognitive radio systems identify pockets of unused spectrum available for transmission, often called spectral holes, without cooperation from any legacy systems operating in the region. Accurate estimation of the locations of the legacy-system transmitters would increase the degree to which cognitive radio nodes could identify and exploit unused spectrum without causing interference.

The localization problem considered here is more challenging than the standard problem of localization in wireless sensor networks because we assume there is no cooperation or feedback with the transmitters. When only one transmitter is present, the transmitter location can be determined from three received power measurements via trilateration, or from a larger number of sensors via a least-squares estimate. However, when there are multiple transmitters contributing unknown proportions of the observed power at each receiver, the non-cooperative localization problem does not admit a straightforward solution.

In related work, Mark and Nasif addressed the transmitter localization problem under the assumption that only one primary transmitter is present in the region of interest [1]. Dogandzic and Amran [2] have derived an EM solution to the single transmitter localization problem under fading and shadowing, but even in that case the solution is highly complex, requiring multivariate numerical integration. Raman et. al conducted an experimental study of transmitter localization performance but deemed localizing multiple transmitters that are simultaneously active too difficult when only signal strength is observed [3].

In earlier work we considered estimating the locations of multiple transmitters in the presence of additive white Gaussian noise (AWGN) [4], [5], which can be considered a special case of the general problem of superimposed signal parameter estimation under additive noise treated by Feder and Weinstein [6]. In this paper, we consider the more realistic lognormal shadowing model, which has been empirically validated to accurately model received power variations due to obstacles in the signal path [7]. As the joint distribution of the hidden and observed random variables in the lognormal model does not produce an analytic EM algorithm, we present a quasi EM algorithm.

II. System Model

Let the unknown two-dimensional locations of the M transmitters be denoted by $\theta = [\theta_1 \;\; \theta_2 \;\; \ldots \;\; \theta_M]^T \in \mathbb{R}^{M\times 2}$, where θ_i is the location of the ith transmitter. We assume that M is known, that all transmitters have the same constant transmit power P_0, and that the locations of the N receivers are known but arbitrary. The problem is then to determine the maximum likelihood (ML) estimate $\hat{\theta}$ of θ based on the observed power measurements at the receivers: ideally, $\hat{\theta} = \arg\max_\theta p(r|\theta)$, where $r = [r_1 \;\; r_2 \;\; \ldots \;\; r_N]^T$ and $r_j \in \mathbb{R}^+$ is the observed power at the jth receiver.

Let $d_j(\theta_i) \in \mathbb{R}^+$ denote the Euclidean distance from the transmitter located at θ_i to the jth receiver. Consider a log-distance path loss model such that the noise-free received power at the jth receiver from the ith transmitter is given by $S_{ij} = \rho P_0 \left(\frac{d_0}{d_j(\theta_i)}\right)^n$, where ρ is a constant that reflects the carrier frequency and antenna properties, n represents the path loss exponent, and d_0 is the close-in reference distance [7].

We assume that lognormal shadowing occurs independently for each transmitter-receiver pair. The resulting unknown measured power from the ith transmitter to the jth receiver is modeled as the random variable $H_{ij} = S_{ij}10^{\frac{X_{ij}}{10}}$, where $X_{ij} \sim \mathcal{N}(0, \sigma^2)$ is the gain in dB due to shadowing. The unknown measurements H_{ij} are related to r_j, the observed

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power at the jth receiver, by

$$r_j = \sum_{i=1}^{M} H_{ij}. \tag{1}$$

Finally, let the set of all $M \times N$ unknown measured powers be $H = [H_1\ H_2 \ldots H_M]$, where $H_i = [H_{i1}\ H_{i2} \ldots H_{iN}]$.

III. Iterative Localization Technique for Lognormal Shadowing

Under independent lognormal shadowing, the likelihood function of H conditioned on transmitter locations θ is the product of MN lognormal densities:

$$p(H = h|\theta) = \prod_{i=1}^{M}\prod_{j=1}^{N} \frac{10\log_{10} e}{h_{ij}\sigma\sqrt{2\pi}} e^{-\frac{(10\log_{10}(h_{ij}) - 10\log_{10}(S_{ij}))^2}{2\sigma^2}}.$$

From (1), the observed power r_j is the sum of M lognormal random variables. No analytic distribution exists for the sum of lognormal random variables, and hence no closed-form expression for $E[H|r, \hat{\theta}]$ can be obtained, which would complicate the expectation step of the EM algorithm.

The following quasi EM approach avoids the conditional likelihood computation required by the true EM approach, and notably does not require any knowledge or estimation of the shadowing variance. The algorithm alternates between (a) estimating each transmitter location independently based on an allocated percentage of the power received at each receiver (Step 4, analogous to the maximization step of the true EM algorithm); and (b) allocating a percentage of the power received at each receiver to each transmitter proportional to the expected received power given the last transmitter location estimates (Steps 2 and 3, analogous to the expectation step of the true EM algorithm). This approach is not limited to lognormal shadowing; in fact, it can be applied to any stochastic model.

STEP 1: Randomly generate initial estimate $\hat{\theta}$ of the M transmitter locations.

STEP 2: Given the current estimate $\hat{\theta}$, determine the expected power in dB at the jth receiver from the ith transmitter for $i = 1$ to M and $j = 1$ to N:

$$\begin{aligned} e_{ij} &= E[10\log_{10} H_{ij}] = E\left[10\log_{10}\left(S_{ij}10^{\frac{X_{ij}}{10}}\right)\right] \\ &= 10\log_{10}(\rho P_0) + 10n\log_{10}\left(\frac{d_0}{d_j(\hat{\theta}_i)}\right). \end{aligned}$$

STEP 3: Normalize the expected values e_{ij} so that they give a total power at each receiver equal to the observed power at that receiver:

$$\tilde{e}_{ij} = 10\log_{10}\left(\frac{r_j 10^{\frac{e_{ij}}{10}}}{\sum_i 10^{\frac{e_{ij}}{10}}}\right).$$

Note that this normalization is proportional, as opposed to the additive normalization prescribed by the EM algorithm for AWGN [5]. We chose to use proportional normalization based on preliminary results and to avoid situations where an additive correction results in a negative power.

STEP 4: Using the expected values $\tilde{e}_{ij}$, re-estimate the transmitter locations by minimizing the sum of squared dB error:

$$\hat{\theta}_i = \arg\min_{\tilde{\theta}_i} \sum_{j=1}^{N}\left(\tilde{e}_{ij} - 10\log_{10}\left(\frac{\rho P_0 d_0^n}{d_j(\tilde{\theta}_i)^n}\right)\right)^2.$$

Note that minimizing the sum of squared dB power error is intuitively pleasing under lognormal shadowing, since the power at receiver j due to transmitter i is a Gaussian random variable in the log domain, and hence squared error is inversely proportional to likelihood.

STEP 5: If the chosen stopping criterion is not yet met, return to Step 2. (In our simulations, we stop after a fixed number of iterations.)

To increase the likelihood that the global minimum of the cost function is reached, the quasi EM algorithm is run to convergence multiple times with different sets of random initial conditions. The final estimate of the transmitter locations is chosen to be the solution that yields the lowest sum-of-squared log-power errors:

$$C(\hat{\theta}) = \sum_{j=1}^{N}\left(\log_{10} r_j - \log_{10}\sum_{i=1}^{M}\left(\frac{\rho P_0 d_0^n}{d_j(\hat{\theta}_i)^n}\right)\right)^2. \tag{2}$$

Figure 1 shows an example of a transmitter-receiver geometry and the associated sum-of-squared-log-power-errors cost function (given in (2)) used to judge possible transmitter locations and select the best estimate. In this example, $M = 2$, and $N = 5$. One transmitter's location is estimated perfectly (to generate a cost function that can be plotted in three dimensions), and the figure shows the cost associated with each possible estimate of the second transmitter's location.

IV. Experiments

We compare the performance of the proposed quasi EM technique to the global optimization method constriction particle swarm optimization (CPSO) [8] and to random guessing. CPSO searches the $(M \times 2)$-dimensional θ-space to directly minimize the sum-of-squared log-power errors cost function (given by (2)) that we use to compare results from different initial conditions for the quasi EM algorithm. For CPSO, we use the inertial weight α, cognitive scaling parameter β, and social scaling parameter γ recommended by Schutte and Groenwold [8].

In our implementation of CPSO, each particle's initial velocity is drawn from a uniform distribution on the interval $[-0.5, 0.5]$ times the width of the search area. One particle's initial location is chosen using the following "smart" technique. First, the k-means algorithm is run to group the N receivers into M clusters based on geographic proximity. Then, we assume that all of the power observed at the receivers in each cluster is generated by a single transmitter, and find the least-squares estimate of that transmitter's location. The resulting set of estimated M transmitter locations constitutes

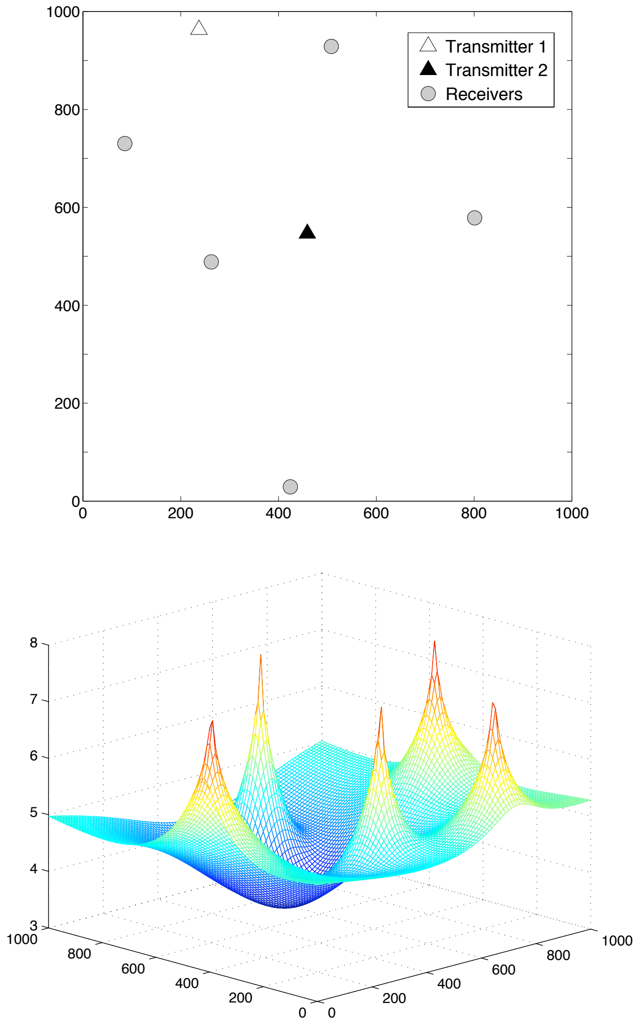

Fig. 1. **Top:** Example scenario for $M = 2$ transmitters and $N = 5$ receivers. **Bottom:** Associated scaled cost surface used to select the best estimate of transmitter 2's location when the location of transmitter 1 is estimated perfectly.

a "smart" initial $(M \times 2)$-dimensional particle in the CPSO search space. The rest of the CPSO initial particles are chosen uniformly and randomly. Preliminary results showing the advantages of using such smart initial guesses rather than random guesses and a more detailed description of these smart initial conditions can be found in [4].

A. Experimental Setup

The region of interest is taken to be a one-kilometer square, with certain constraints on the transmitter and receiver geometries. First, transmitters are assumed to be separated by at least 200 meters, reflecting the physical reality that primary transmitters using the same frequency band would interfere if they were too close together. Second, all receivers are assumed to be at least twice the reference distance $d_0 = 1$ from all transmitters, a constraint required to guarantee that the log-distance propagation model yields realistic results [7].

These placement constraints are reflected in the implementation of the proposed quasi EM algorithm. While the algorithm is iterating, if any transmitter location estimates are within 100 meters of each other, or if any single estimate is more than 50% (500 meters) outside the square region of interest, the algorithm is assumed to be converging incorrectly and the problematic transmitter(s) are uniformly randomly reassigned to new locations in the square region. Once the algorithm stops iterating, any location estimates that lie outside the square region are clipped to its nearest boundary. In contrast, the locations of the CPSO particles are clipped every iteration in order to keep the particles from collecting outside the search space which makes the search defunct.

The quasi EM algorithm is run M^2 times with different uniformly randomly drawn initial estimates $\hat{\theta}$; the number of iterations for each initial estimate is fixed at 10. Because the quasi EM algorithm does not implement the precise expectation and maximization steps of the true EM algorithm, it is not guaranteed to converge to a local minimum of the likelihood function, and theoretical analysis of its convergence behavior is difficult if not impossible. In simulation, we found that the quasi EM algorithm typically converges within 10 iterations. To allow fair comparison with competing schemes, we chose to perform a fixed number of iterations rather than implementing a convergence-based stopping criterion. In our simulations, CPSO uses M^2 particles, one of which is generated via the previously described "smart" technique, and is allowed 10 iterations per particle for a total of $10M^2$ guesses. Similarly, the random guessing approach makes $10M^2$ random guesses (uniformly drawn from the search space) of the transmitter locations and chooses the random guess that minimizes the sum-of-squared log-power error cost function given in (2).

B. Results

The simulated performance of the quasi EM, CPSO, and random guessing algorithms is presented in Figs. 2 and 3. The chosen performance metric is the average squared distance error between estimated and true transmitter locations, where the average is taken over the M transmitters. The values have been normalized to assume a square of unit area. Performance figures show the median error over 1000 different random draws for $M = 2$ and $M = 3$ transmitters, $N = 2M$ to $N = 40$ receivers, and shadowing variance of $\sigma^2 = 4$ and $\sigma^2 = 16$.

Fig. 2 presents simulation results for $M = 2$ transmitters. The quasi EM localization approach produces the smallest median error across all values of N and both values of σ^2 considered, and the performance gap increases as the number of receivers moves from $N = 2M$ to $N = 20$. As expected, performance error for all three localization techniques initially decreases with increasing N but eventually flattens; as the number of receivers grows large, the power measurement provided by each additional receiver is less likely to provide independent information, and hence the resulting performance improvements are not significant. The performance advantage of the quasi EM approach is significantly larger when M is increased to 3 transmitters, as shown in Fig. 3. In fact, for $N = 40$ receivers and the lower noise case (Fig. 3 top), the quasi EM approach achieves a 10-fold reduction in error relative to CPSO and a roughly 50-fold reduction relative to random guessing.

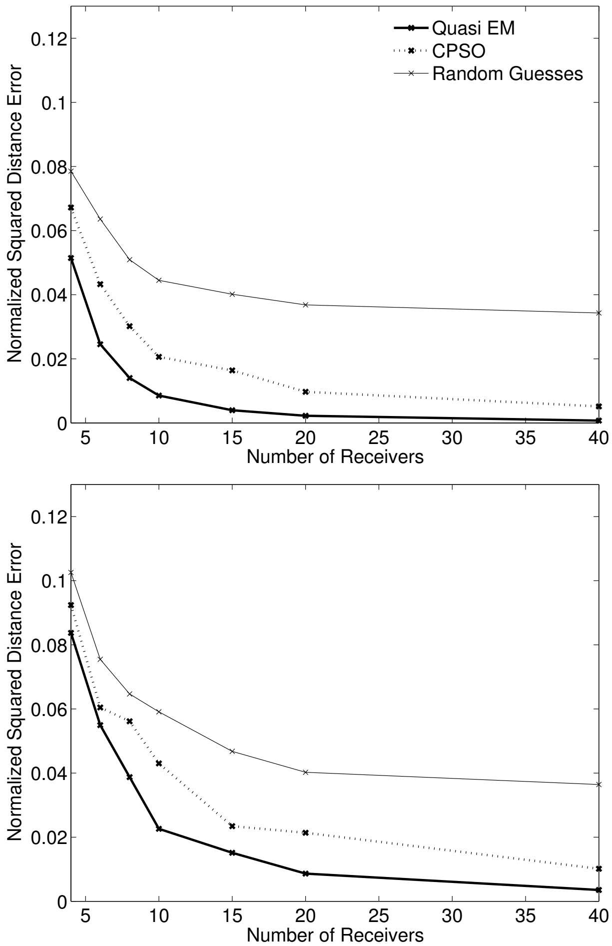

Fig. 2. Two transmitters: Median normalized average squared distance error for $\sigma^2 = 4$ (top) and $\sigma^2 = 16$ (bottom).

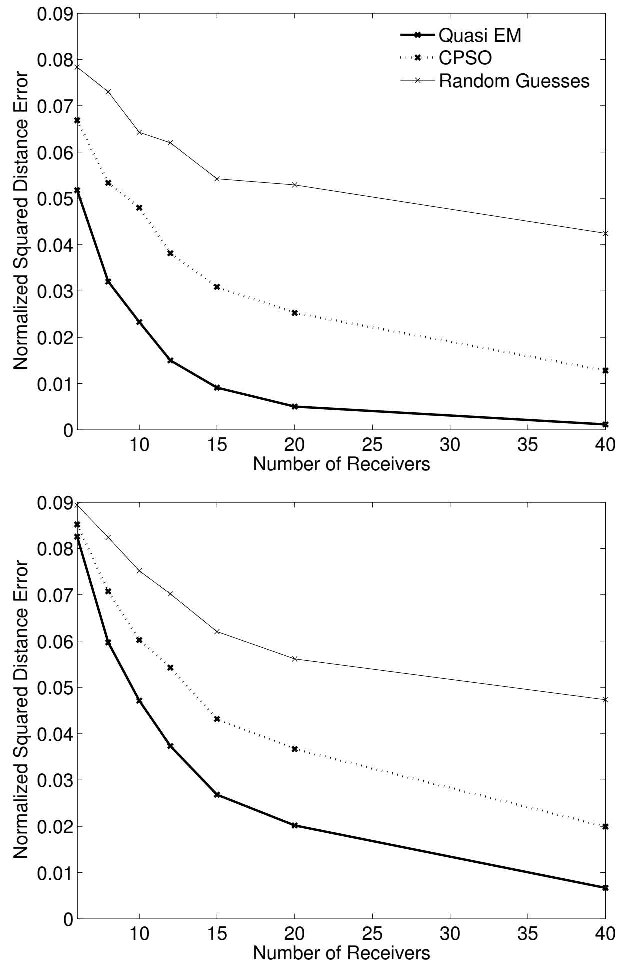

Fig. 3. Three transmitters: Median normalized average squared distance error for $\sigma^2 = 4$ (top) and $\sigma^2 = 16$ (bottom).

V. Conclusions and Open Questions

We have shown that a quasi EM approach can be effective in estimating multiple transmitter locations from multiple power measurements when propagating signals experience lognormal shadowing. Compared to a state-of-the-art global optimization method and to random guessing, the proposed quasi EM algorithm achieves a significant reduction in error when given the same number of guesses.

Although we focused on lognormal shadowing, other propagation models might also benefit from this quasi EM approach, which alternates between assigning each receiver some responsibility for locating a transmitter and using the receiver locations and their assigned responsibilities to estimate where the transmitters might be. Also, in this work, we use the sum of log-power errors as a proxy objective for the function we would actually like to minimize: the sum of distances between the estimated and true transmitter locations. It may be that a more effective proxy cost function exists, and this may indeed depend on the path loss and noise models.

In the investigation and results presented here, we assume that the number of transmitters is known and that each transmits at a known power. How to effectively extend the estimation algorithms to model an unknown number of transmitters and unknown transmit powers is an open question.

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