JOINT DECONVOLUTION AND CLASSIFICATION FOR SIGNALS WITH MULTIPATH

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ABSTRACT

For many sensing modalities such as sonar, received signals may be corrupted by multipath that degrades performance in classification. An approach to jointly deconvolve and classify such signals is proposed, in which training data is used to increase the overall system performance. Specifically, a filter is estimated that minimizes the distortion between the received signal and a set of training signals, then the received signal is assigned to the class that corresponds to the training signal whose estimated filter is most sparse. Simulations compare the new method with blind deconvolution using Cabrelli’s algorithm followed by a correlation-based nearest neighbor classifier. Results indicate that joint deconvolution and classification performs similarly to blind deconvolution at low and moderate noise levels.

Index Terms—deconvolution, multipath channels, sonar signal processing, sonar target recognition, pattern classification

1. INTRODUCTION

Consider the problem of automatically classifying signals that have experienced unknown multipath distortion. This is a common problem in sonar, and arises in other sensing modalities such as ultrasound, THz, and radar. One solution is to classify based on features designed to be invariant to multipath. Another solution is to perform blind deconvolution and then classify on the recovered signal. We propose a third solution, joint deconvolution and classification, and demonstrate that it can be a more effective system-optimized approach than blind deconvolution.

Assume there exists a set of training signals \( \{x_i\} \) for \( i = 1, \ldots, M \), and a corresponding classification label \( y_i \) for each signal. A received signal \( z(t) \) is known to be contaminated with multipath and additive noise,

\[
z(t) = h(t) \ast c(t) + w(t),
\]

where \( c(t) \) is the unknown “clean” direct path signal of interest, \( h(t) \) is the impulse response of a linear time-invariant (LTI) filter corresponding to the multipath effect, and \( w(t) \) is additive noise. The problem is to classify \( z(t) \) as \( y \).

One approach to classifying in the presence of multipath is to extract classification features from the training and received signals that are (ideally) invariant to multipath distortion. Shin et al. consider a number of time-frequency features for clutter rejection [1]. Strausberger et al. compare different distance measures for 1-nearest-neighbor classification of radar signals passed through Rician channels [2]. Other work in this area includes [3].

A second category of solutions is to perform blind deconvolution of each received signal to attempt to remove the multipath distortion, then pass the cleaned signal to a classifier (which may then extract pertinent classification features). There are many examples of trying to remove the multipath by blind deconvolution in order to classify [3–11]. Roan et al. report good experimental results with blind deconvolution for an acoustic PVC pipe filter [11]. Also, Broadhead and Pflug report excellent correlation scores for blindly deconvolved signals distorted by multipath [5]. They show that Cabrelli’s method [12] outperforms Wiggins’ deconvolution [10] in the presence of additive Gaussian white noise. However, classification is not considered in [5].

In the following section, a new approach is proposed that avoids blind deconvolution by using training signals to jointly characterize the multipath channel and classify the received signal.

2. JOINT DECONVOLUTION AND CLASSIFICATION

When considered jointly with multipath distortion, a new approach to nearest-neighbor classification is to ask: could the received signal \( z(t) \) be from the same class as training signal \( x_i(t) \), but appear different due to some multipath distortion \( h_i(t) \)? A straightforward nearest-neighbor formulation of this question is to classify \( z \) as the class \( \hat{y} = y_i \ast \) where

\[
i^* = \arg \min_{i=1, \ldots, M} D \left( z(t) - x_i(t) \ast \hat{h}_i(t) \right), \tag{1}
\]

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where the $\hat{h}_i(t)$ are constrained to be multipath filters and $D$ is an appropriate distance measure such as the $L^2$ norm. Solving (1) requires a strict definition of what constitutes a multipath filter, and then the minimization must be solved.

A related, but simpler way to implement this idea is to solve (1) in two steps: relax the multipath constraints when estimating $\hat{h}_i(t)$ for $i = 1, \ldots, M$, then choose $i^*$ such that $\hat{h}_{i^*}(t)$ is most like a multipath filter. To do this, we first estimate $\hat{h}_i(t)$ for each of the training signals:

$$\hat{h}_i(t) = \frac{Z(w)}{X_i(w)},$$

(2)

where capital letters denote the Fourier transform of the corresponding time-domain signals. Assuming a perfect deconvolution is possible, the distortion between $z(t)$ and each $x_i(t) * \hat{h}_i(t)$ is zero, so that it is not possible to apply a nearest-neighbor classifier using the estimated $\hat{h}_i(t)$ as per (1). However, given the assumption that the received signal underwent multipath, we can classify based on which estimated deconvolution filter best fits a multipath criterion. That is, classify $z(t)$ as $\hat{y} = y_{i^*}$ where

$$i^* = \arg \max_{i=1, \ldots, M} F(\hat{h}_i(t)),$$

(3)

and $F$ is a functional that evaluates how well the input argument represents a multipath filter.

This opens the question of what is an appropriate functional $F$. A similar problem of evaluating how well a filter represents a multipath filter occurs in blind deconvolution. Blind deconvolution of multipath for sonar and geophysical sensing have previously modeled multipath filtering as a few paths of non-frequency-selective reflections, so that a multipath filter is well-represented by a few time-shifted and scaled impulses. Given this model it is common to use the sparseness of $\hat{h}(t)$ as criteria for a multipath filter. Metrics for quantifying sparseness are specified by Wiggins [10], Cabrelli [12], Broadhead [5], and others.

### 3. Experiments

We evaluate joint deconvolution and classification as specified in (2) and (3) by comparing its performance with blind deconvolution using Cabrelli’s method [12] followed by classification. In Experiment 1, we compare the ability to classify received multipath signals using standard statistical learning assumptions of independent and identically distributed training and testing signals. In Experiment 2, we build on the work by Broadhead and Pflug [5] by comparing the classification results of signals corrupted by multipath and additive white noise.

Both simulations are two-class problems and all signals are discrete-time sequences. For consistency, sparsity of an estimated filter $\hat{h}_i$ is evaluated using the D-norm criterion specified by Cabrelli’s blind deconvolution [12]. Thus (3) becomes

$$i^* = \arg \max_{i=1, \ldots, M} \| \hat{h}_i \|_\infty$$

For simplicity, the length of each $\hat{h}_i$ is fixed to the true length of the multipath distortion so that the D-norm does not favor shorter impulse responses. In our experiments, we remove the mean of the estimated filter prior to computing the D-norm. In both approaches, a straight inverse in the Fourier domain is used for any deconvolution operations.

#### 3.1. Experiment 1

Experiment 1 uses two sonar signals $x_a$ and $x_b$, shown in Fig. 1, that were randomly selected echoes from different clutter collected with an impulsive-source active sonar system. Both signals have been normalized such that their standard deviation is unity. Training signals $x_i$ are generated independently and identically by first randomly choosing class one or class two with equal probability. If $x_i$ is chosen to be from class one, then $x_i = x_a + w_i$, where $w_i$ is drawn from a zero-mean Gaussian distribution with standard deviation $\sigma$. Similarly, if $x_j$ is selected to be from class two, then $x_j = x_b + w_j$. Thus, signals in class one are perturbed versions of $x_a$, and signals in class two are perturbed versions of $x_b$, where the extent of perturbation is described by $\sigma$.

![Fig. 1](image-url)
a randomly generated multi-pass filter,

\[ h_j = \sum_{k=1}^{K} \alpha_k \delta(t - \tau_k) \]

where \( \tau_k \) are chosen at random uniformly over the interval [0, 400], and the corresponding magnitudes of \( \alpha_k \) decrease exponentially with \( t \). To allow for direct path and multipath interference, the sign of \( \alpha_1 \) is positive, while the signs of the remaining \( \alpha_2, \ldots, \alpha_K \) are chosen randomly. In our simulations, we choose \( K \) randomly as an integer in the interval [1, 6]. The resulting test signal \( z_j = h_j \ast c_j \) is then passed to the two methods for classification.

Nearest-neighbor classification using correlation to define nearness is performed after the blind deconvolution. First, we estimate a pre-filtered signal \( \hat{c}_j \) for each \( z_j \) using Cabrelli’s method, and compare \( \hat{c}_j \) to each training sequence \( x_i \) for \( i = 1, \ldots, M \). The classification label \( \hat{g}_i \) for \( \hat{c}_j \) is chosen to be \( g_i \), where \( i^* \) maximizes the max normalized correlation (also used as a metric by Broadhead and Pflug [5]),

\[ i^* = \arg \max_{i=1, \ldots, M} \frac{\max |x_i \otimes \hat{c}_j|}{\| x_i \| \| \hat{c}_j \|}, \]

where \( \otimes \) denotes correlation.

The simulation was run with \( M = 10 \) training sequences and \( N = 100 \) testing signals for \( \sigma \) ranging over the interval [0, 1.5]. The performance comparison of the two methods is shown in Fig. 2. Joint deconvolution and classification performs well through perturbations of \( \sigma = 1 \), but drops to the performance of blind deconvolution as large perturbations in the signals render the classes more ambiguous.

![Fig. 2](image_url)  
**Fig. 2.** Percentage of correct classification as a function of \( \sigma \) for the joint deconvolution/classification and blind deconvolution methods compared in Experiment 1.

### 3.2. Experiment 2

A second experiment uses the same training signals and experimental architecture as the blind deconvolution experiment in [5] to test the sensitivity to additive noise, but is augmented with an additional classification step. In this setup, we simulate the classification of structured signals \( x_g \) and \( x_s \) in Fig. 3 after they have been corrupted by a random multipath filter \( h_j \) and additive noise \( w_j \) such that \( z = x_g \ast h_j + w_j \), if the test signal is drawn randomly from class one, or \( z = x_s \ast h_j + w_j \) if drawn from class two, and \( w_j \) is drawn from a zero-mean Gaussian distribution with varying standard deviation, as to adjust the signal to noise ratio (SNR) of \( z \).

![Fig. 3](image_url)  
**Fig. 3.** Structured signals used to generate training and testing data for class one (above) and class two (below) in Experiment 2.

As done in [5], we employ a simple bandpass filter prior to classification since it is reasonable that in practice, some knowledge of the frequency characteristics would be available. For blind deconvolution, the filter is applied to the estimated signal \( \hat{c}_j \) retrieved from Cabrelli’s method. For joint deconvolution and classification, it is applied to the test signal \( z_j \) prior to application of (2).

We classify \( N = 1000 \) test signals for each SNR level ranging from 0 to 30 dB, as shown in Fig. 4. Performance of the two algorithms at very low SNR is comparable (performance for both methods below 0 dB is around 50%), whereas joint deconvolution and classification outperforms blind deconvolution for moderate and high SNR levels.

![Fig. 4](image_url)  
**Fig. 4.** Percentage of correct classification as a function of SNR for the joint deconvolution/classification and blind deconvolution methods compared in Experiment 2.
4. DISCUSSION AND OPEN QUESTIONS

We have proposed joint deconvolution and classification as a system-optimized alternative to blind deconvolution followed by classification. As a benchmark for performance, we employed Cabrelli’s method [12] for blind deconvolution, followed by classification based on normalized correlation as reported in [5]. Results indicate that joint deconvolution and classification is comparable to blind deconvolution at very low SNR, and outperforms blind deconvolution at moderate and high SNR. Improvement in classification performance is a result of the system-optimized approach of the joint method.

Since blind deconvolution requires a matrix inversion for each test signal, its computational complexity increases exponentially with the length $n$ of the test signal (Levinson recursion is $O(n^2)$). This is an additional expense over the joint method since the computation complexity of classification in the two methods is similar. Using our MATLAB implementation for Experiment 2, the blind deconvolution and classification was four orders of magnitude slower than joint deconvolution and classification. This included naive optimizations in matrix inversion during blind convolution, such as incorporating Levinson recursion in a compiled MEX function for inverting the Toeplitz matrix [12].

While we have demonstrated one implementation of joint deconvolution and classification, there may be other such system-optimized approaches to this problem. For example, a viable approach would be to estimate a filter for each pairing of test and training signals, but to constrain the choice of the estimated filter $\hat{h}(t)$ to represent the expected multipath (rather than choose the most sparse of the estimated filters, as we did). This problem could be reduced to parameter estimation on a multipath filter model, such as incorporating Levinson recursion in a compiled MEX function for inverting the Toeplitz matrix [12].

In this paper the signal samples were used as features for nearest neighbor classification, but in many tasks there are known features that more readily differentiate the classes. For such cases, it is an open question how to jointly remove multipath distortion and classify. Also, we have focused on the case of multipath as a common type of distortion. The proposed technique might work well for general LTI smoothing if a suitable metric can be found to replace $F$ in (3) for evaluating the particular “smooth” criteria. Lastly, in some applications, multipath distortion is time-varying over the signal duration. For that case, joint signal retrieval and classification methods require further investigation.

5. REFERENCES